Towards a Neural Game Theory

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1 Some puzzles

- Why is it that approximately 50% of people vote?
- Why is Bitcoin worth money?

2 Toy model of voting

Imagine that we have n voters and k agent fragments, with $n \gg k$.

Let us denote the agent fragment values by $x \in \mathbb{R}^k$.

We posit the following model, which for reasons that will become obvious we call the Brightside Model. Let eager_i be the eagerness of agent i to vote, expressed as a probability that the agent will vote. Let opinion_i be the opinion of the agent, expressed on an axis from 1 (reasonable) to -1 (shitty).

$$\text{eager}_i = \sigma(Ax + a)$$
$$\text{opinion}_i = \tanh(Bx + b)$$

We then have that the expected net vote is

$$\mathbb{E}\left[\text{vote}_{\text{net}}\right] = \sum_i \text{eager}_i \cdot \text{sign}(\text{opinion}_i)$$

and the expected outcome of the election is

$$\text{outcome} = \text{sign}\left(\text{vote}_{\text{net}}\right)$$

and the expected reward of agent i is then

$$\mathbb{E}\left[r_i\right] = -\text{eager}_i + 10 \cdot \text{opinion}_i \cdot \text{outcome}$$

We then consider the following problem:

$$\max_{A_i, a_i} \min_x \mathbb{E}\left[r_i\right]$$

where A_i, a_i refer to the portions of A and a that are under the control of agent i

We find that optimizing this objective via block gradient descent produces populations of agents that vote with probability that often settles near 50 %.

Note that we use a differentiable variant of the sign function:

$$\text{sign}(\theta) = \tanh(\theta/\epsilon)$$

where ϵ is chosen to be $1e-6$.

3 Toy model of the stock market

TODO. Note that simulations are currently producing prices that sometimes spiral off to huge values or tiny values; how do we incorporate real-world limits on prices?

4 Simulation results

- If we assign random values to x rather than minimizing the expected reward with respect to x, almost nobody votes.
- If we maximize the expected reward with respect to x, very few people vote.
- If we minimize the expected reward with respect to x, an average of 39.6% people vote.

5 Theoretical considerations

5.1 What does x represent?

x seems to represent "what everyone knows" or "social facts". That is, things that have no existence independent of the minds of the agents, but which nevertheless have an existence that cannot be willed away by any single agent.

5.2 Why minimize over x?

It sort of makes sense if you think about it. We are maximizing our expected reward given the contents of everyone else's heads; we are pessimistic about the contents of everyone else's heads. It is thus a fairly standard application of the minimax rule: act so that you will do well given the worst-case value of every option.

5.3 How do people know the values they would need to know?

We suspect that there are some simple sufficient statistics that people could plausibly know. Need to calculate the gradients by hand to check this out though.

$$\begin{aligned}
\frac{\partial \mathbb{E}\left[r_i\right]}{\partial x} &= -\frac{\partial \text{eager}_i}{\partial x} + 10 \cdot \frac{\partial \text{opinion}_i}{\partial x} \cdot \text{outcome} + 10 \cdot \text{opinion}_i \cdot \frac{\partial \text{outcome}}{\partial x} \\
&= -\text{eager}_i \left(1 - \text{eager}_i\right) A_i + 10 \cdot \left(1 - \text{opinion}_i^2\right) \cdot \text{outcome} \\
&\qquad + 10 \cdot \text{opinion}_i \cdot \left(1 - \frac{\text{outcome}^2}{\epsilon^2}\right) \frac{\partial \text{vote}_{\text{net}}}{\partial x}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \text{vote}_{\text{net}}}{\partial x} &= \sum_j \left[\frac{\partial \text{eager}_j}{\partial x} \cdot \text{sign}\left(\text{opinion}_j\right) + \text{eager}_j \cdot \frac{\partial \text{sign}\left(\text{opinion}_j\right)}{\partial x}\right] \\
&= \sum_j \left[\text{eager}_j (1 - \text{eager}_j) A_j \cdot \text{sign}\left(\text{opinion}_j\right) + \text{eager}_j \cdot \left(1 - \frac{\text{opinion}_j^2}{\epsilon^2}\right) B_j\right]
\end{aligned}$$

5.4 But people have free will!

The voting model is consistent with free will, since there is a free bias parameter for every agent. Every agent is thus free to adjust that parameter so that they almost always vote or almost always don't vote. Nevertheless, having to pick a behavior other than the default behavior is more difficult and therefore less likely in our simulations.